Chimera States in Nonlinear Systems with Multiple Delayed Feedbacks

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Delays affect your life in a number of ways

Long-distance light propagation

Propagation delays in the brain

Traffic jams

Hot-cold water control in a shower!
Delayed-feedback Systems
Delay Differential Equations (DDEs)

\[ \tau \dot{x}(t) + x(t) = \beta f (x(t - \tau_D)) \]

- \( x(t) \) is dynamical variable
- \( \tau_D \) is delay time
- \( \tau \ll \tau_D \) is system response time
- \( f(x) \) is nonlinear transformation
- \( \beta \) is feedback gain
Dynamical regimes. Bifurcations

\[ \beta = 1.9 \]

\[ \beta = 2.2 \]

\[ \beta = 2.55 \]
Applications of NL Delay Dynamics

- Chaos communications, 1995
- High spectral purity microwave oscillators, 1994
- Photonic Reservoir Computing, 2012
- Chimera study in networks of virtual oscillators, 2013
Understanding NL Dynamical Networks is Crucial

- Power grids
- Internet
- Social networks
- Bird swarms
- Brain

* Image from S. H. Strogatz (Nature, 2001.)
What are chimeras?

...monstrous fire-breathing creatures
Chimeras: Kuramoto and Delay Networks

- Chimera is a network state consisting of incongruent parts: **coherent and chaotic**.

- Chimera states can arise in many real-world networks:
  - Power grids
  - Networks of neurons in the human heart

Leading to failure of the system.

**Are chimeras possible in delay systems?**

*Y. Kuramoto (Nonlin. Phenomena in Complex Sys., 2002.)*
Chimeras in single delay systems
Experimental Setup For Delay Chimeras

- Laser light is NL transformed
- The signal is delayed, filtered
- The signal is modulating the wavelength of the laser
Experimental Setup For Delay Chimeras

\[ \varepsilon \dot{x} + \delta \int_{t_0}^{t} x(\xi) d\xi + x = \beta f(x(t-1)) \]
Experimental Setup For Delay Chimeras

\[ \varepsilon \dot{x} + \delta \int_{t_0}^{t} x(\xi) d\xi + x = \beta f \left( x(t - 1) \right) \]
System Properties

**NL function asymmetry**

Bistability: low gradient and large gradient

Singular limit map $x_n = \beta f(x_{n-1})$
"Coupling" Induced By Filters

\[ \varepsilon \dot{x} + \delta \int_{t_0}^{t} x(\xi) d\xi + x = \beta f(x(t-1)) \]

\[ x_{\sigma}(n') = x_{\sigma}(n' - 1) + \int_{\sigma - 1}^{\sigma} h(\sigma - \xi') \cdot f [x_{\xi'}(n' - 1)] d\xi' \]

Impulse response function \( h(t) \)

Discrete coupling

Continuous coupling

Nonlinearity
DDE "Coupling": Impulse Response Function

Long-range coupling thanks to the integral term

$$\delta \int_{t_0}^{t} x(\xi) \, d\xi$$

$$x_\sigma(n') = x_\sigma(n' - 1) + \int_{\sigma-1}^{\sigma} h(\sigma - \xi') \cdot f \left[ x_{\xi'}(n' - 1) \right] d\xi'.$$

- Discrete coupling
- Impulse response
- Continuous coupling
- Nonlinearity
Network Analysis: Space-Time Representation

Two-dimensional representation of a delayed dynamical system

F. T. Arecchi, * G. Giacomelli, A. Lapucci, and R. Meucci
Istituto Nazionale di Ottica, Largo E. Fermi 6, 50125 Firenze, Italy
(Received 31 July 1991; revised manuscript received 10 December 1991)

- Stacking temporal coordinates
- Virtual space
Network Analogy. Space-Time Representation

\[ x(t) \]

\[ 0 \quad \tau_D \quad 2\tau_D \quad 3\tau_D \quad t \]

\[ 0 \quad \tau_D \quad 2\tau_D \quad 3\tau_D \quad 4\tau_D \]

\[ 0 \quad \tau_D \quad 2\tau_D \quad 3\tau_D \quad 4\tau_D \]

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Theory vs Experiment

- First demonstration of chimeras in delay systems


- Exist over long time
- Excellent agreement between the model and experiment
Multiheaded Chimeras

- Tunable number of heads
- Increased system complexity
- Coexistence of chimeras with different number of heads
- Can be possibly used in applications
Double delay systems
System with two delays

\[ i_{DBR_{\text{in}}} \rightarrow i_{DBR} \rightarrow i_{\text{act}} \rightarrow \text{Fabry-Pérot slab} \rightarrow P_0 \rightarrow P(\lambda) \rightarrow \text{Photodiode} \]

\[ \rightarrow i_x \rightarrow \text{Bandpass filter} \rightarrow i_D \rightarrow \tau_1 \rightarrow \text{Delay line} \rightarrow \tau_2 \gg \tau_1 \rightarrow \text{Delay line} \]
System with two delays

\[ \varepsilon \dot{x} + \delta \int_{t_0}^t x(\xi) d\xi + x = (1 - \gamma) f(x(t - 1)) + \gamma f(x(t - 100)) \]
Coherent core
Incoherent core
Dissipative solitons: optical memory medium?
Vectorial dissipative solitons in vertical-cavity surface-emitting lasers with delays

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Regenerative memory in time-delayed neuromorphic photonic resonators

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\( x \)

Time

\( 2\tau_2 \)

\( 0 \)

\( 4\tau_1 \)

\( -0.1 \)

\( 0.7 \)

\( x \)

-0.1

0.7

\( 0 \)

\( 2\tau_2 \)

\( 0 \)

\( 4\tau_1 \)
Multistability

Brunner et al. arxiv:1712.03283
Take-away message

The dynamical behavior on the delay interval can be translated to a network.

Those networks allow observation of chimera states and dissipative solitons.
Applications of Chimera States/Dissipative solitons

Study of synchronization in complex networks:
  - Power grids
  - Networks of neurons in the human heart

Neuromorphic computing
  - Optical memory
Thank You

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